**Exam**
**Mechanics & Relativity 2017–2018**
**December 15, 2017**

INSTRUCTIONS

- This exam comprises 3 problems. Start your solution of each problem on a separate sheet.
- Write the name of your tutor and/or group on the top right-hand corner of the first sheet handed in, and *drop your work in the box with your tutor's name.* [*]
- The answers to problems 1 through 3 require clear arguments and derivations, all written in a well-readable manner.
- The number of points for every subquestion are indicated inside a box in the margin. The total number of points per problem is

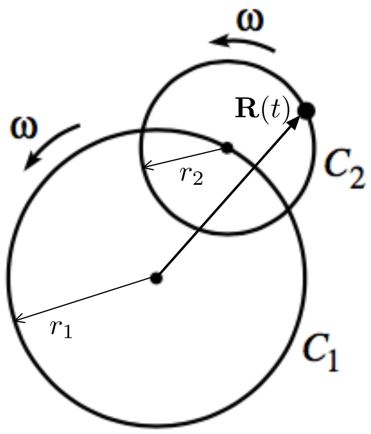
| Problem | # of points |
|---|---|
| 1 | 5 |
| 2 | 10 |
| 3 | 5 |

  and the grade is computed as (total # points) $/20 * 9 + 1$.

[*]Anupam: is this still useful in the current grading procedure?

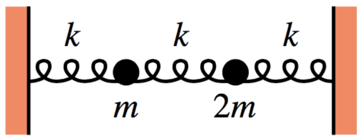
*The answers to problems 1 through 3 require clear arguments and derivations, all written in a well-readable manner.*

**Problem 1** Two circles in a plane, C1 and C2, each rotate with frequency $\omega$ (as measured relative to an inertia frame). The center of C1 is fixed in an inertial frame. The center of C2 is fixed on C1. A mass $m$ is fixed on C2.




The position of the mass relative to the center of C1 is indicated by the vector $\mathbf{R}(t)$. Find the direction and the magnitude of the fictitious force felt by the mass due to rotations of both circles. 5

**Problem 2** Three identical springs and two masses, $m$ and $2m$, lie between two walls as shown in Figure below. The positions of the left-hand and right-hand masses are $x_1(t)$ and $x_2(t)$, respectively, relative to their equilibrium positions.




a. Show that the potential energy of the system can be written as: 1

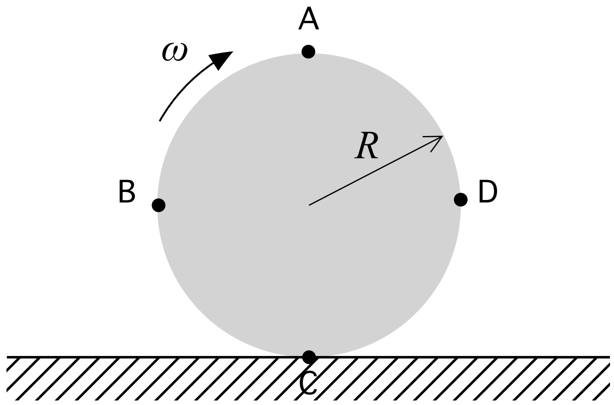
$$V = k({x_1}^2 - x_1 x_2 + {x_2}^2)$$

b. Derive the equations of motion for the two masses. 2

c. Find the eigenmodes, and show that the eigenfrequencies are 7

$$\sqrt{\frac{k}{m}}\sqrt{\frac{3 \pm \sqrt{3}}{2}}$$

**Problem 3** Consider a disk of radius $R$ rolling without slipping on the ground. While it spins with angular velocity $\omega$ its forward velocity has magnitude $v = R\omega$. At a particular instant we mark 4 points on the circumference, A thru D, with C being the point that is currently in contact with the ground. Since the disk does not slip, the instantaneous velocity of C is zero.




a. Copy the schematic and add the spin *vector* $\boldsymbol{\omega}$. [1]

b. Show that the velocity vector of the center of the disk satisfies the vectorial relation $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$ in which $\mathbf{r}$ is the position vector from the contact point C to the center. [1]

c. Use this expression to determine the velocity vectors of the points A, B and D. [3]